INVESTIGATION OF THERMOCAPILLARY CONVECTION IN A TWO-LAYER FLUID

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We examine the problem of thermocapillary motion in a system of two immiscible fluids with a linear temperature distribution at the bottom. To a first approximation in small Marangoni number we obtain a solution for velocity, temperature, and pressure in each layer.

Knowledge of the principles of thermal convection in layered liquid systems is of interest for understanding the hydrodynamics and heat- and mass-exchange processes in the application of multi-layer coverings.

Investigation of thermocapillary convection in systems of several immiscible fluid layers has been stimulated, on the one hand, by the development of methods which intensify convective fluid mixing under the influence of thermocapillary convection (chemical technology), and on the other, by the search for methods which suppress convective mixing (material science in outer space) by means of suitable selection of configuration and parameters of the fluid layers [1, 2].

The exact solution obtained here for temperature and pressure allows us to reveal interesting features of thermocapillary motion in a two-layered system. It can also serve as a standard solution for the verification of numerical computer programs.

We consider the motion in a system of two plane layers of viscous, incompressible fluids of thicknesses H_1 and H_2, respectively (Fig. 1). We assume that the fluids do not intermix and that the fluid density of the upper layer is less than that of the lower ($\rho_1 > \rho_2$). The lower boundary of the system is a solid surface. We consider that a constant linear temperature distribution is maintained at the rigid boundary. At the fluid–fluid boundary a tangential thermocapillary stress acts, due to the nonuniform temperature distribution. The coefficient of surface tension depends on temperature according to the nonlinear law

$$\sigma_1 = \sigma_{01} + \frac{1}{2}\alpha_1 (T_1 - T_0)^2;\ \sigma_{01} = \text{const},\ \alpha_1 = \text{const}.$$

Here T_0 is the value of the temperature corresponding to the extremal value of the coefficient of surface tension.

The thermocapillary motion with such a dependency has previously been studied in [3, 4].

We seek the distributions of temperature and velocity in each of the fluid layers when the upper boundary of the system is: 1) a solid surface; and 2) a free surface at which

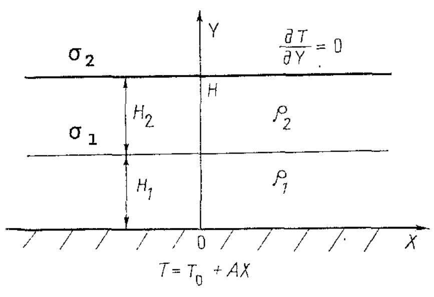

Fig. 1. Problem geometry.

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the surface tension depends on temperature according to the nonlinear law

$$\sigma_2 = \sigma_{02} + \frac{1}{2}\alpha_2 (T_2 - T_0)^2;\ \sigma_{02} = \text{const},\ \alpha_2 = \text{const}.$$

Due to the thermal conductivity of the fluid, the temperature distribution at the fluid–fluid boundary and the free surface will be nonuniform. This gives rise to tangential thermocapillary stresses and induces motion in the fluid layers. We examine a steady regime of such motion, when the tangential thermocapillary motion is balanced by the action of the viscous forces.

Using the standard simplifying assumptions, the mathematical formulation of the problem includes the Navier–Stokes equations and the equations of heat conduction and continuity:

$$U_i \frac{\partial U_i}{\partial X} + V_i \frac{\partial U_i}{\partial Y} = -\frac{1}{\rho_i}\frac{\partial P_i}{\partial X} + \nu_i \Delta U_i,$$

$$U_i \frac{\partial V_i}{\partial X} + V_i \frac{\partial V_i}{\partial Y} = -\frac{1}{\rho_i}\frac{\partial P_i}{\partial Y} + \nu_i \Delta V_i - g,\quad \frac{\partial U_i}{\partial X} + \frac{\partial V_i}{\partial Y} = 0, \tag{1}$$

$$U_i \frac{\partial T_i}{\partial X} + V_i \frac{\partial T_i}{\partial Y} = \chi_i \Delta T_i.$$

At the solid surface (Y = 0), we prescribe the nonslip condition and maintain a constant linear temperature distribution

$$U_1 = V_1 = 0,\ T_1 = T_0 + AX,\ A = \text{const}.$$

At the boundary separating the fluids $Y = H_1$, we prescribe: temperature and velocity continuity

$$U_1 = U_2,\ T_1 = T_2,$$

the condition of impenetrability

$$V_1 = V_2 = 0,$$

heat flux continuity

$$k_1 \frac{\partial T_1}{\partial Y} = k_2 \frac{\partial T_2}{\partial Y},$$

and balance of viscous forces

$$\eta_1 \frac{\partial U_1}{\partial Y} = \eta_2 \frac{\partial U_2}{\partial Y} + \frac{d\sigma_1}{dT_1}\frac{\partial T_1}{\partial X}.$$

At the upper boundary Y = H, we have:

1) in the solid surface case, the nonslip condition $U_2 = V_2 = 0$, thermal insulation of the surface $\partial T_2/\partial Y = 0$;

2) in the free surface case, the balance of viscous and thermocapillary forces

$$\eta_2 \frac{\partial U_2}{\partial Y} = \frac{\partial \sigma_2}{dT_2}\frac{\partial T_2}{\partial X};$$

nondeformation of the surface, $V_2 = 0$, and thermal insulation, $\partial T_2/\partial Y = 0$.

As will be shown below, the assumption of surface planarity is approximately fulfilled for heavy fluids, and for the action of sufficiently large thermocapillary pressure we have σ_{01}, σ_{02} large.

We introduce the dimensionless parameters and variables:

$$x = \frac{X}{H},\ y = \frac{Y}{H},\ h_1 = \frac{H_1}{H},\ h_2 = \frac{H_2}{H},\ h = \frac{h_1}{h_2},$$

$$\Pr_i = \frac{\nu_i}{\chi_i} \text{ — the Prandtl number.}$$

We will seek a self-similar solution of the form

$$U_i = \frac{\nu_i}{H} x\psi_i'(y),\ V_i = -\frac{\nu_i}{H}\psi_i(y),\quad T_i = T_0 + AHx\Theta_i(y),$$
$$P_i = P_{0i} - \rho_i gHy - \frac{1}{2}\rho_i\left(\frac{\nu_i}{H}\right)^2(\lambda_i x^2 + f_i(y)), \tag{2}$$

where $P_{01} = P_1(0, 0) = \text{const}$ and $P_{02} = P_2(0, 1) = \text{const}$ are the pressures at the lower and upper boundaries, respectively.

A self-similar solution to the problem of motion in a fluid layer was found in [5, 6]. In order to detemrine the unknown functions $\psi_i(y)$, $\Theta_i(y)$, $f_i(y)$ and the constants λ_i, we obtain from (1), (2) the following two-point boundary-value problem for the nonlinear system of ordinary differential equations:

$$\psi_i''' + \psi_i\psi_i'' - (\psi_i')^2 + \lambda_i = 0,\ 1/2f_i' = \psi_i\psi_i' + \psi_i'',$$
$$\Theta_i'' - \mathrm{Pr}_i(\psi_i'\Theta_i - \psi_i\Theta_i') = 0 \tag{3}$$

with the conditions at the bottom

$$\psi_1(0) = \psi_1'(0) = 0,\ \Theta_1(0) = 1,$$

and at the upper boundary $\Theta_2'(1) = 0$:

1) for the solid boundary case

$$\psi_2(1) = \psi_2'(1) = 0;$$

2) for the free surface case

$$\psi_2 = 0,\ \psi_2''(1) = m_2\Theta_2^2(1),$$

where

$$m_2 = \frac{\alpha_2 A^2 H^3}{\eta_2\nu_2}.$$

At the interface $y = h_1$ we have: $\psi_1' = \nu\psi_2'$, $\psi_1 = \psi_2 = 0$, $\Theta_1 = \Theta_2$, $\Theta_1' = k\Theta_2'$, $\psi_1'' - \eta\nu\psi_2'' = m_1\Theta_1^2$, where $\nu = \nu_2/\nu_1$; $\eta = \eta_2/\eta_1$; $k = k_2/k_1$; $m_1 = \alpha_1 A^2H^3/(\eta_1\nu_1)$ is the Marangoni number at the boundary between the fluids.

In order to reveal the characteristic features of thermocapillary flow, we obtain an approximate analytical solution in each of the layers for small values of the Marangoni number, assuming that the Prandtl number is of the order of unity.

In the free-surface case at the upper boundary there exist two Marangoni numbers this problem, and there is the possibility of carrying out an expansion in either of them. For generality of exposition, we will carry out the expansion in the interphase Marangoni number m_1, assuming that both numbers are equal in order of magnitude.

For $m_1 = m_2 = 0$, the problem has the solution

$$\psi_i = 0,\ f_i = 0,\ \lambda_i = 0,\ \Theta_i = 1, \tag{4}$$

which corresponds to the fluid at rest with a uniform temperature distribution with height.

For $|m_i| < 1$, using the method of small perturbations we will construct a solution in the form

$$\psi_i = m_1\psi_i^{(1)} + m_1^2\psi_i^{(2)} + \ldots,\quad f_i = m_1f_i^{(1)} + m_1^2f_i^{(2)} + \ldots,$$
$$\lambda_i = m_1\lambda_i^{(1)} + \ldots,\ \Theta_i = 1 + m_1\Theta_i^{(1)} + \ldots \tag{5}$$

Substituting (5) in (3) and neglecting terms quadratic in m_1, we obtain

$$\psi_i''' + \lambda_i = 0,\ f_i' = 2\psi_i'',\ \mathrm{Pr}_i\psi_i' = \Theta_i''. \tag{6}$$

To avoid confusion with the layer number, here and below we omit the upper index of 1 denoting the first term of the expansion.

The boundary conditions:

$$\psi_1(0)=\psi_1'(0)=0,\ f_1(0)=f_2(1)=0,\ \Theta_1(0)=0,\ \Theta_2'(1)=0, \tag{7}$$

1) $\psi_2(1) = \psi_2'(1) = 0$ (solid boundary);

2) $\psi_2(1) = 0$, $\psi_2''(1) = m_2/m_1$ (free surface);

$$m_2/m_1=\alpha/(\eta\nu),\ \alpha=\alpha_2/\alpha_1.$$

At the interface $y = h_1$:

$$\psi_1(h_1)=\psi_2(h_1)=0,\ \ \psi_1'(h_1)=\nu\psi_2'(h_1),\ \psi_1''(h_1)-\eta\nu\psi_2''(h_1)=1,$$
$$\Theta_1(h_1)=\Theta_2(h_1),\ \Theta_1'(h_1)=k\Theta_2'(h_1).$$

Solving the boundary-value problem (6) with conditions (7), we find the fundamental values λ_i and correspondingly the first terms of the expansion (5).

1. Rigid Upper Boundary Case. With an accuracy up to terms of order $O(m_1{}^2)$, from (4)-(7) we obtain for the velocity, temperature and pressure fields

$$\lambda_1=-\frac{3}{2}\frac{h}{h_1(\eta+h)},\ \ \lambda_2=-\frac{3}{2}\frac{1}{\nu h_2(\eta+h)},$$
$$V_1=\frac{m_1\nu_1}{H}\frac{\lambda_1}{6}y^2(y-h_1),\ \ V_2=\frac{m_1\nu_2}{H}\frac{\lambda_2}{6}(y-1)^2(y-h_1),$$
$$U_1=-\frac{m_1x\nu_1}{H}\frac{\lambda_1}{2}y\left(y-\frac{2}{3}h_1\right),$$
$$U_2=-\frac{m_1x\nu_2}{H}\frac{\lambda_2}{2}(y-1)\left(y-\frac{2h_1+1}{3}\right),$$
$$P_1=P_{01}-\rho_1gHy+\frac{m_1\rho_2\nu_1^2}{2H^2}\lambda_1\left(y^2-x^2-\frac{2}{3}h_1y\right), \tag{8}$$
$$P_2=P_{02}-\rho_2gHy+\frac{m_1\rho_2\nu_2^2}{2H^2}\lambda_2\left(y^2-x^2-\frac{2h_1+4}{3}y+\frac{2h_1+1}{3}\right),$$
$$T_1=T_0+AHx\left[1-\frac{m_1\,\mathrm{Pr}_1\lambda_1}{72}y^3(3y-4h_1)\right],$$
$$T_2=T_0+AHx\left\{1-\frac{m_1}{72}[\mathrm{Pr}_2\lambda_2(y-h_1)^2(3(y-h_1)^2-8h_2(y-h_1)+6h_2^2)-\mathrm{Pr}_1\lambda_1h_1^4]\right\}.$$

This case corresponds to the motion of a two-layer fluid in the gap between plates when heated from below. Figure 2 shows the flow lines and the profile of the horizontal velocity component in the region $x > 0$ in the case where the layers are of equal thickness $h_1 = h_2 = 0.5$. Since the boundary conditions for the velocity are the same for both layers and from (4) it follows that for Marangoni number $m_1 = 0$ the vertical distribution of temperature is uniform, then to a first order in m_1 we obtain a flow pattern which is symmetric with respect to the interface boundary $y = 0.5$. Substituting the values of λ_i and m_1 into (8), we obtain the following estimate for the velocity:

$$V_i(y)\sim 1/(\eta_1h_2+\eta_2h_1),\ U_i(y)\sim 1/(\eta_1h_2+\eta_2h_1).$$

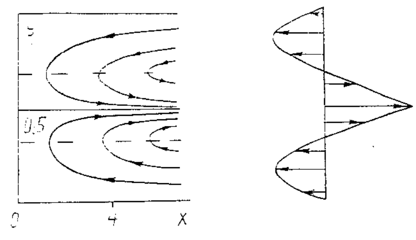

Fig. 2. Flow lines and profile for the horizontal component of the velocity in the region x > 0, for $h_1 = h_2 = 0.5$. The upper boundary is solid.

Consequently, the value of the viscosity influences only the scale of the velocity, and does not destroy the symmetry of the flow. In this approximation, temperature deviation from the initial profile does not affect the velocity field, the dependence on χ appearing in the next higher order in m_1.

2. Free Surface Upper Boundary. In this case the solution to problem (6), (7) has the form

$$\lambda_1 = \frac{3h(\alpha-2)}{h_1(3\eta+4h)},\quad \lambda_2 = -\frac{3(\eta+\alpha(\eta+2h))}{\nu\eta h_2(3\eta+4h)},$$

$$V_1 = \frac{m_1\nu_1}{H}\frac{\lambda_1}{6}y^2(y-h_1),\quad V_2 = \frac{m_2\nu_2}{H}\frac{\lambda_2}{6}(y-1)(y-h_1)(y-a),$$

$$U_1 = -\frac{m_1\nu_1 x}{H}\frac{\lambda_1}{2}y\left(y-\frac{2}{3}h_1\right),\quad U_2 = -\frac{m_1\nu_2 x}{H}\frac{\lambda_2}{2}(y-b_+)(y-b_-),$$

$$P_1 = P_{01} - \rho_1 gHy + \frac{m_1\rho_1\nu_1^2}{2H^2}\lambda_1\left(y^2-x^2-\frac{2}{3}h_1 y\right), \tag{9}$$

$$P_2 = P_{02} - \rho_2 gHy + \frac{m_1\rho_2\nu_2^2}{2H^2}\lambda_2\left(y^2-x^2-\frac{2}{3}(a+h_1+1)y+\right.$$

$$\left.+\frac{2}{3}\left(a+h_1-\frac{1}{2}\right)\right),\quad T_1 = T_0 + AHx\left\{1-\frac{m_1 \mathrm{Pr}_1\lambda_1}{72}y^3(3y-4h_1)\right\},$$

$$T_2 = T_0 + AHx\left\{1-\frac{m_1}{72}[\mathrm{Pr}_2\lambda_2(y-h_1^2)(3(y-h_1)^2+4(y-h_1)(h_1-h_2-a)+6h_2(a-h_1))-\lambda_1\mathrm{Pr}_1 h_1^4]\right\},$$

where

$$a = \frac{\eta(1+h_2)+\alpha(\eta(h_1-h_2)+2h_2)}{\eta+\alpha(\eta+2h)};$$

$$b_\pm = \frac{a+h_1+1}{3} \pm \frac{1}{3}\sqrt{1+a^2+h_1^2-a-h_1-ah_1}.$$

In our problem, the normal stresses at the interface boundary and at the free surface are not constant, and this leads to distortion of these surfaces. We will consider that the distortion is eliminated due to the large value of g; in this case as follows from (8) and (9), the pressure is primarily hydrostatic.

In both this and the previous case, to first order in m_1 the velocity distribution does not depend on the temperature profile distortion, that is, on χ or Pr. From the expressions for U_i and V_i in (9) it follows that it is possible to have different regimes of stationary flow in the two-layer system, depending on the parameters α and η and the thickness of the layers. We can distinguish three regimes:

regime I

$$\alpha < \alpha^* = \frac{\eta}{2(\eta+h)},$$

$$a>1,\ b_+>1,\ h_1<b_-<1,\ V_1>0,\ V_2<0;$$

regime II

$$\alpha^* < \alpha < 2,$$

$$h_1<a<1,\ h_1<b_\pm<1,\ V_1>0,\ V_2 \text{ changes sign};$$

regime III

$$\alpha > 2,$$

$$a<h_1,\ b_-<h_1,\ h_1<b_+<1,\ V_1<0,\ V_2>0.$$

Figure 3 shows flow lines for different regimes for η = 1/3 and equal layer thicknesses $h_1 = h_2 = 0.5$.

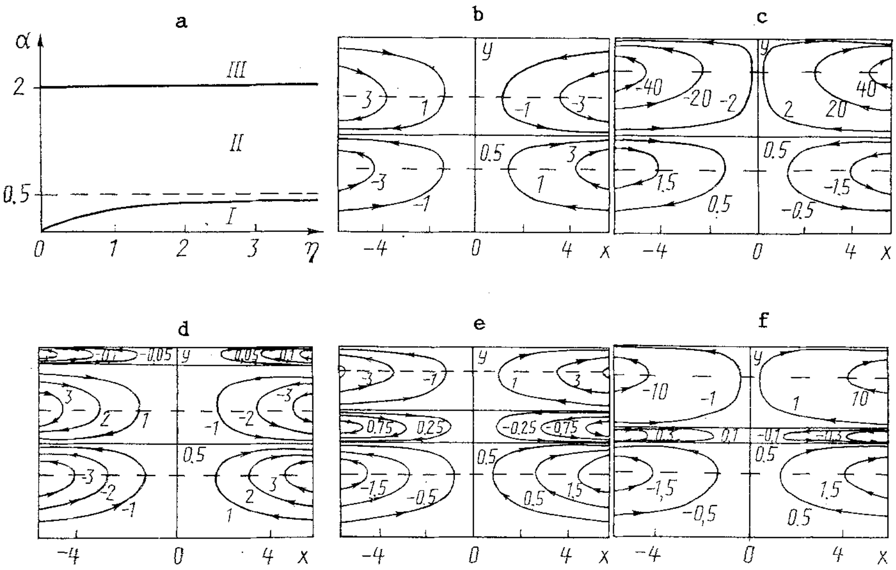

Fig. 3. Flow regimes in the case where the upper boundary is a free surface: a) the bounds of the regimes as a function of the parameters α and η; b-f) flow lines for $\eta = 1/3$, $h_1 = h_2$; b) regime I, $\alpha = 1/16$; c) regime II, $\alpha = 1/6$; d) regime II, $\alpha = 1/2$; e) regime II, $\alpha = 1$; f) regime III, $\alpha = 3$. The values of the stream function (numbers on the curves) are given in units of $10^{-2}\ \nu_1 m_1$.

For small α corresponding to the first regime $\alpha < \alpha^*$ (Fig. 3b, $\alpha = 1/16$), the surface tension between the fluids is much larger than at the free surface. The intensity of motion in both layers is of the same order, the direction of the vortices and the flow pattern are similar to the rigid upper boundary case.

In the second regime, $\alpha^* < \alpha < 2$, there is a competition between motions engendered by the Marangoni forces at the boundary between fluids and at the free surface (Fig. 3d-f). For sufficiently small α close to α^*, another vortex with direction opposite that of the circulation arises near the surface (Fig. 3d), and motion in the lower layer is diminished. With subsequent growth in α (Fig. 3e-f) the dimensions of the vortex at the free surface and the magnitude of the circulation both increase, the middle vortex contracts with a reduction in strength. As a result the vortical motion velocity drops in the lower layer.

We should single out the value $\alpha = 2$. In this case, as is evident from (9), the lower fluid is motionless, while in the upper there is quite intense single-vortex motion, the structure of the flow not depending on the ratio of the layer thicknesses. This means that by covering the working fluid with another fluid of certain properties, it is possible to either decrease or suppress thermocapillary convection in the lower layer.

To interpret our solution (9), we must consider the ratio of the thickness of the upper layer to the lower $h = h_2/h_1$ to be greater than or of the order of m_1.

In the third regime $\alpha > 2$ (Fig. 3c) the motion is determined by the Marangoni forces at the free surface. The direction of vortical motion in the lower fluid changes to the opposite direction, and the intensity of the motion in the upper layer is considerably greater than in the previous cases.

In all regimes the fluid flow in the lower layer changes direction for $y = 2h_1/3$.

The influence of the ratio of viscosities η on the fluid motion is weaker than that of the parameter $\alpha = \alpha_2/\alpha_1$ (Fig. 3a). For $\alpha > 1/2$, a change in η does not lead to a change in flow regime, but merely influences the position of the separation boundary between vortices and the turning plane of the fluid in the upper layer. For $0 < \alpha < 1/2$, the transition from the first regime to the second occurs for $\eta \geq 2\alpha h(1 - 2\alpha)$.

Figure 4 shows the distribution of the horizontal component of the velocity with height for both the case of equal (Fig. 4a) and different (Fig. 4b) layer thicknesses. We see that the structure of the flow is unchanged by changes in h_1. The intensity of the flow in the

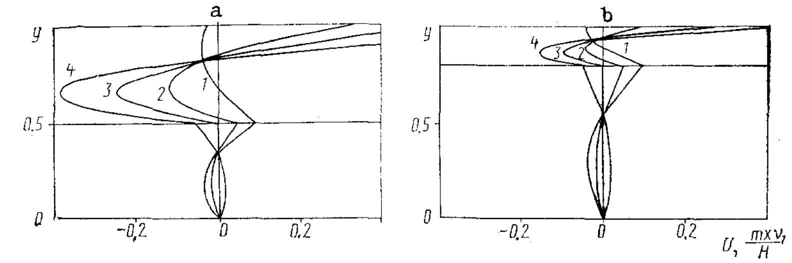

Fig. 4. Distribution of the horizontal component of the velocity with depth for various ratios of layer thicknesses. a) $h_2/h_1 = 1$; b) $h_2/h_1 = 0.25$. Curves 1-4 correspond to $\alpha = 1/16$, 1, 2, 3, respectively.

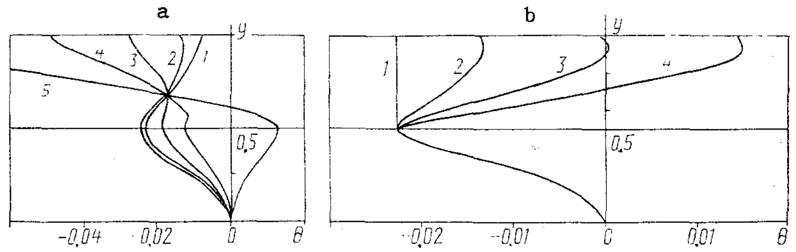

Fig. 5. Distribution of the deviation of temperature θ with depth for $\eta = 1/3$: a) $\chi = 2$, various flow regimes; curves 1-5 correspond to $\alpha = 1/16$ (regime I), 1/6, 1/2, 1 (regime II), 3 (regime III), respectively; b) various ratios of the thermal diffusion coefficients $\chi = \chi_2/\chi_1$ for fixed Pr_1, $\alpha = 1/6$ (regime II); curves 1-4 correspond to $\chi = \infty$, 2, 0.825, 0.5, respectively.

lower layer undergoes insignificant changes and in the upper layer in regimes II and III (curves 2-4) the intensity decreases with increasing thickness of the lower layer h_1. This is because the competition of motions engendered by the Marangoni forces occurs in a smaller volume and as a result, the motions strongly inhibit one another.

Figure 5a shows the deviation of the temperature from the linear profile

$$\theta = \frac{12(T_i - (T_0 + AHx))}{m_1 \mathrm{Pr}_1 AHx}, \quad i = 1,\ 2.$$

It is evident from a comparison of Fig. 5a and Fig. 3 that the extrema of $\theta(y)$ correspond to the boundaries separating vortices — regions in which the intensity of the motion is maximum.

We will examine in more detail the variation of the temperature profile with depth in regimes I and II (curves 1-4, Fig. 5a). Since the prescribed temperature gradient is maintained at the bottom, the temperature rises with increasing distance from the center $x > 0$. Fluid elements from the bottom layer are drawn in by the vortex and are carried to the colder central region, lose heat and are returned to the boundary separating the fluids with lower temperature than that prescribed by the initial distribution. The circulation is directed in the opposite direction in the upper layer, the fluid is carried into the hot region, is heated and returned to the boundary separating the vortices (or to the surface, in the case of regime I) with a higher temperature than at the boundary between layers. This temperature value is the local maximum temperature in the fluid. In the presence of a third vortex (regime II) in the near-surface layer heat transfer occurs as in the lower layer, and the temperature drops. In regime III, where the motion is determined by the Marangoni forces at the free surface, the direction of the fluid circulation is opposite. Consequently, the fluid is heated in proportion to its approach to the boundary separation (curve 5) and cools when receding.

Figure 5b gives the temperature deviation $\theta(y)$ for various $\chi = \chi_2/\chi_1$. For $\chi \to \infty$ we observe a uniform temperature distribution in the upper fluid layer (curve 1). The magnitude of the deviation from its initial value ($T_0 + AHx$) is determined by the temperature at the separation boundary. For decreasing χ the influence of thermal conductivity is reduced in comparison with that of the convective transfer of heat.

NOTATION

X, Y, x, y are dimensional and dimensionless Cartesian coordiantes; H_i, h_i are dimensional and dimensionless thicknesses of the i-th fluid layer; $H = H_1 + H_2$ is the total thickness of the fluid layers; $h = h_2/h_1$ is the ratio of the upper layer thickness to the lower; U_i, V_i are the horizontal and vertical velocity components; P_i, T_i are the pressure and temperature in the i-th fluid layer, respectively; $\sigma_i = \sigma_{0i} + 1/2\alpha_i (T_i - T_0)^2$ is the coefficient of surface tension; T_0 is the temperature corresponding to the extremum value of the coefficient of surface tension; σ_{0i} is the extremum of σ_i; α_i is a coefficient in the σ_i dependence; $\alpha = \alpha_2/\alpha_1$ is the ratio of these coefficients; α^* is the value of α corresponding to the boundary between regimes I and II; A is the coefficient of the temperature dependence at the lower boundary; g is the acceleration of gravity; ρ_i is the density of the i-th fluid layer; P_{01}, P_{02} are the pressures on the axis of symmetry at the lower and upper boundaries of the system; ν_i, η_i, χ_i, k_i are the coefficients of kinematic and dynamic viscosity, thermal diffusivity and thermal conductivity; $\nu = \nu_2/\nu_1$, $\eta = \eta_2/\eta_1$, $\chi = \chi_2/\chi_1$, $k = k_2/k_1$ are the ratios of the coefficients of kinematic and dynamic viscosity, thermal diffusivity and thermal conductivity of the upper and lower layers; $Pr_i = \nu_i/\chi_i$ is the Prandtl number; $m_i = \alpha_i A^2 H^3/(\eta_i \nu_i)$ is the Marangoni number; $\theta = \frac{12(T_i T_0 - AHx)}{m_1 \mathrm{Pr}_1 AHx}$ is the deviation of the fluid temperature from the linear profile. The lower index i = 1 denotes the variable and coefficient values for the lower layer, i = 2 denotes those for the upper layer; the upper index $^{(1)}$ indicates the first term of an expansion; ' denotes differentiation with respect to y.

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